USE OF THE SCHLIEREN METHOD TO INVESTIGATE TEMPERATURE FIELDS IN A SOLID

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The use of the Schlieren method to measure the temperature field and temperature gradients in a solid is considered. The results obtained are compared with a theoretical calculation of the temperature field in a finite cylinder for boundary conditions of the second and third kind.

Shadow methods are widely used to investigate inhomogeneities in gases and liquids [1]. These methods can also be used to investigate inhomogeneities in solids.

In this paper we consider the problems involved in making a quantitative investigation of optical inhomogeneities caused by the temperature field in a solid in the form of a cylinder. One of the important advantages of the Töpler method or the Schlieren method is the fact that it enables one to observe visually the dynamics of the development of the temperature field both of stationary and moving heat sources, and when suitably calibrated, quantitative calculations can also be carried out. Using photography, one can record the state of the temperature field at any instant of time. The method is based on the change in the angle of deviation of a beam of light by optical inhomogeneities in a solid, due to inhomogeneity of the temperature field.

The apparatus was assembled using the parallel beam arrangement and a Ronk grating on an OSK-3 optical bench. It enabled us to measure the angle of deviation of the light beam with an accuracy up to $2 \cdot 10^{-4}$ radian.

The sensitivity of the apparatus can be improved considerably if one uses a Foucault knife edge as the visualizing diaphragm. To make quantitative measurements it is necessary to establish in advance the relation between the angle of deviation of the light beam and the temperature gradient, i.e., to calibrate the apparatus.

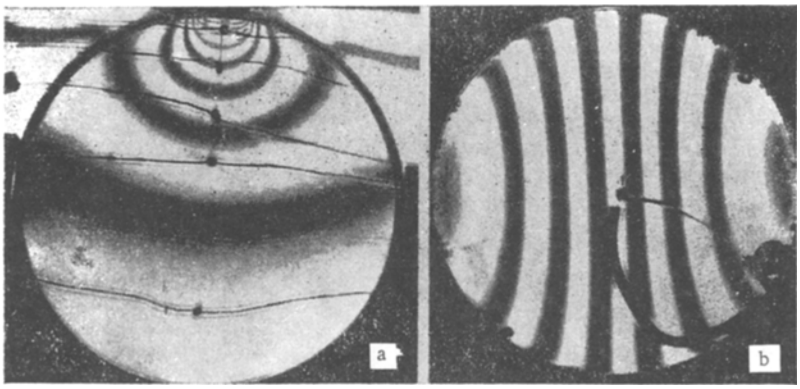

Fig. 1. Calibration of the Schlieren (a) and a Schlieren photograph of the distribution of the horizontal isogradients in a finite cylinder, uniformly heated over the whole side surface (b).

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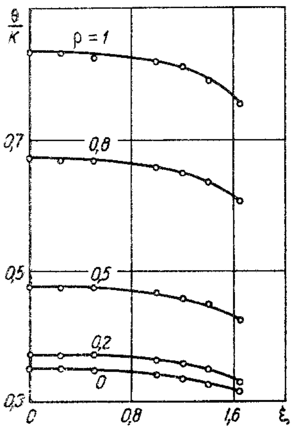

Fig. 2. Graph of the variation of $\theta(\xi)/K$.

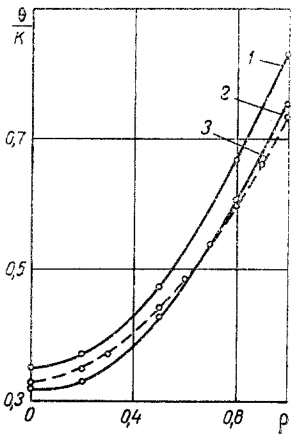

Fig. 3. Graph of the variation of $\theta(\rho)/K$: 1) for the center of the finite cylinder $\xi = 0$; 2) for the end $\xi = 1.645$; 3) results obtained by the Schlieren method.

Calibration is carried out as follows [2]. Over the end of the cylinder chromel–kopel thermocouples are attached along the radius. Using an ÉPP-09 electronic potentiometer we measured the temperature and simultaneously photographed the distribution pattern of the bands of temperature isogradients (Fig. 1a). The coordinates of the points at which the thermocouples are attached were found using a KM-6 cathetometer. From the results obtained we constructed a graph of the temperature distribution $T = T(r)$, from which we found the value of each isogradient $\partial T/\partial r$. The isogradients were calculated up to the heater. As the zero band we took that part of the specimen in which there were no Schlieren within the limits of the resolving power of the method. We will compare the experimental data obtained by calibration with theoretical calculations. To do this we will consider the temperature field in a finite circular cylinder of radius R and length $2l_0$, uniformly heated along the side surface by a heat flux $q = q(t)$. Heat transfer takes place through the end of the cylinder from a medium at a temperature T_0. The initial temperature of the cylinder is T_0. We will find the temperature distribution for $t > 0$. The problem can be reduced to the solution of the heat conduction equation (in dimensionless quantities)

$$\frac{\partial\theta}{\partial \mathrm{Fo}} = \frac{1}{\rho}\cdot\frac{\partial}{\partial\rho}\left(\rho\,\frac{\partial\theta}{\partial\rho}\right) + \frac{\partial^2\theta}{\partial\xi^2} + P(\mathrm{Fo}) \tag{1}$$

with the initial and boundary conditions

$$\theta(\rho,\ \xi,\ 0) = 0, \qquad \left.\frac{\partial\theta}{\partial\rho}\right|_{\rho=1} = 0, \tag{2}$$

$$\left.\frac{\partial\theta}{\partial\xi}\right|_{\xi=-l} = -\mathrm{Bi}\,\theta|_{\xi=l}, \qquad \left.\frac{\partial\theta}{\partial\xi}\right|_{\xi=l} = \mathrm{Bi}\,\theta|_{\xi=-l}. \tag{3}$$

The function P(Fo) represents the thermal source, and can be represented in the form

$$P(\mathrm{Fo}) = \frac{2K(\mathrm{Fo})}{1-\rho_1^2}\,\delta(1-\rho_1)\,\delta(\xi \pm l), \tag{4}$$

where $1 - \rho_1 = \varepsilon$; ε is an arbitrary small positive quantity,

$$\delta(1-\rho_1) = \begin{cases} 0 & \rho < \rho_1, \\ 1 & \rho_1 \leqslant \rho \leqslant 1, \end{cases}$$

$$\delta(\xi \pm l) = \begin{cases} 0 & |\xi| > l, \\ 1 & |\xi| \leqslant l. \end{cases}$$

Omitting the calculations here, we will write the final solution obtained using Fourier and Hankel transformations

$$\theta = \sum_k B_k \cos k\xi \sum_s \frac{2J_0(s\rho)}{J_0(s)} \int_0^{\mathrm{Fo}} \exp\left[-(s^2+k^2)(\mathrm{Fo}-\tau)\right] K(\tau)\,d\tau, \tag{5}$$

where summation with respect to k is carried out over all the roots of the equation

$$k \sin kl - \mathrm{Bi}\cos kl = 0, \tag{6}$$

while summation with respect to s is carried out for all roots of the equation

$$J_0'(s) = 0.$$

In the case of a constant thermal flux $K(\mathrm{Fo}) = K = \mathrm{const}$, Eq. (5) takes the form

$$\theta = 2K \sum_k B_k \cos k\xi \sum_s \frac{J_0(s\rho)}{J_0(s)\,(s^2+k^2)} \left\{1 - \exp\left[-(s^2+k^2)\,\mathrm{Fo}\right]\right\}. \tag{7}$$

Using Eq. (7) we calculated the Schlieren (Fig. 1b) for Fo = 0.3, Bi = 0.42 [3], and $q/\lambda = \partial T/\partial r$, equal to $1.41 \cdot 10^4$ degree/m. The change in temperature along the axis of the cylinder from the center to the end for different values of ρ ($0 \le \rho \le 1$) is shown in Fig. 2. The change in temperature from the axis to the periphery at the center of the cylinder and at its end is shown in Fig. 3. Curve 1 shows the temperature distribution in the center of the cylinder $\xi = 0$. Curve 1 agrees with the similar calculation for an infinite cylinder [4], within the limits of calculational error. Curve 2 shows the temperature distribution at the ends of the cylinder ($\xi = \pm 1.645$). The dashed curves show values of the temperature found by the Schlieren method with subsequent graphical integration. Good agreement is observed between the curves in the range $\rho = 0.60$–0.85, and a small disagreement between the theoretical and experimental points close to the axis and the surface of the cylinder. This may be due to the fact that the theoretical calculation involves the Biot criterion, the value of which is taken for the average integral temperature over the end of the cylinder. Analysis of the relative error in the isogradients $\partial T/\partial r$ shows that the greatest relative error does not exceed 5%. At the point of intersection of the curves (Fig. 3) the spread between the theoretical and experimental values of the temperature is 2.35%.

In conclusion we note that the above method can be used to determine the thermal characteristics of a solid and their temperature dependence.

With the appropriate calibration, we can easily determine

$$\frac{q}{\lambda} = \left.\frac{\partial T}{\partial r}\right|_{r=R},$$

whence, from the known value of the thermal flux q, we can find the thermal conductivity λ for the given temperature. From the well-known dependence of the refractive index on the temperature and on the mechanical stresses we can also make quantitative investigations of the field of the temperature stresses.

NOTATION

$2l_0$	is the length of the cylinder;
n	is the refractive index;
T and T_0	are the absolute temperature of the cylinder and the medium;
$\theta = (T - T_0)/T_0$	is the dimensionless temperature;
R	is the radius of the cylinder;
$\rho = r/R$	is the dimensionless radius;
a and λ	are the thermal diffusivity and the thermal conductivity;
t	is the time;
$Fo = at/R^2$	is the Fourier number;
$\xi = z/R$	is a dimensionless coordinate;
$l = l_0/R$	is the dimensionless half-length;
q(Fo)	is the thermal flux density;
$K(Fo) = (R/\lambda T_0)q(Fo)$	is a dimensionless complex;
$P(Fo) = [2K(Fo)/(1-\rho_1^2)]\delta(1-\rho_1)\delta(\xi \pm l)$	is a dimensionless complex;
$Bi = \alpha R/\lambda$	is the Biot number;
$J_0(s)$	is the Bessel function of the first kind and zeroth order.

LITERATURE CITED

1. L. A. Vasil'ev, Shadow Methods [in Russian], Nauka, Moscow (1968).
2. V. I. Shakhurdin, in: Proceedings of the Scientific Conference of the Perm Polytechnical Institute, Perm Polytechnical Institute Press, Perm (1968), p. 31.
3. Z. K. Yang and R. B. Otting, Heat Transfer, Collection of translations, Mir, 1 (1969), p. 163.
4. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).
5. E. A. Chistova, Tables of Bessel Functions of the Real Argument and Their Integrals [in Russian], Akad. Nauk SSSR, Moscow (1958).
6. E. A. Chistova, Tables of Zeros of Bessel Functions [in Russian], Akad. Nauk SSSR, Moscow (1967).